Previous Paper (Solved)

University of Delhi

B.TECH – Information Technology and Mathematical Innovations (IT & M)

Entrance Exam-2018*

1. The number of ways in which an examiner can assign 30 marks to 8 questions, giving not less than 2 marks to any question, is:

   A. ${}^{21}C_9$ B. ${}^{21}C_{10}$

   C. ${}^{21}C_8$ D. ${}^{21}C_7$

2. A circle passes through the points (–1, –1) and (3, 7). Which of the following cannot also be a point on the circle?

   A. (0, 1) B. (1, 1)

   C. (1, 0) D. (0, 0)

3. A circle is tangent to both negative x axis and positive y axis and passes through the points (–4, 6). The sum of the radii of all such possible circles is:

   A. 24 B. 12

   C. 20 D. 10

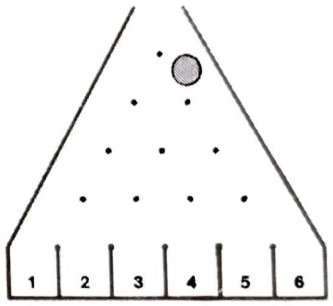
4. A circle is inscribed in an ellipse. If 'P' is the probability that a point within the ellipse chosen at random lies outside the circle, then the eccentricity of the ellipse is:

   A. $\sqrt{1-P^2}$ B. $\sqrt{1-P}$

   C. $\sqrt{(1+P)^2-1}$ D. $\sqrt{1-(1-P)^2}$

5. The figure shows a peg board used for a game show. Whenever the gray chip falls onto a peg, it has an equal chance to bounce either to the left or the right peg right.

   If the chip is dropped from the right peg on the second row (as shown), the slot it would most likely fall to is:



   A. 6 B. 1

   C. 3 D. 4

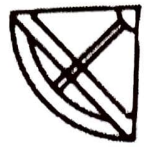
6. A man takes a forward step with probability 0.8 and backward step with probability 0.2. What is the probability that at the end of 9 steps he is exactly three steps away from the starting point:

   A. $\frac{5377}{5^7}$ B. $\frac{5354}{5^6}$

   C. $\frac{5376}{5^7}$ D. $\frac{69888}{5^8}$

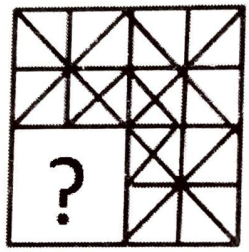
7. In each of the following questions a part of the figure is missing. Find the right figure from the options to fit the missing pieces:

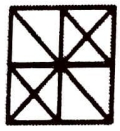
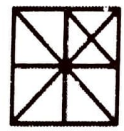
* Exam was held on 21/06/2018.

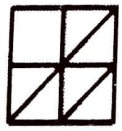
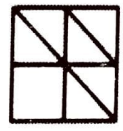
A. B. 

C. D.

8. In each of the following questions a part of the figure is missing. Find the right figure from the options to fit the missing pieces.



A.  B. 

C.  D. 

9. Let $f(x) = ax^2 + bx + c$ and $f(-1) < 1$, $f(1) > -1$, $f(3) < -4$ and $a \neq 0$, then

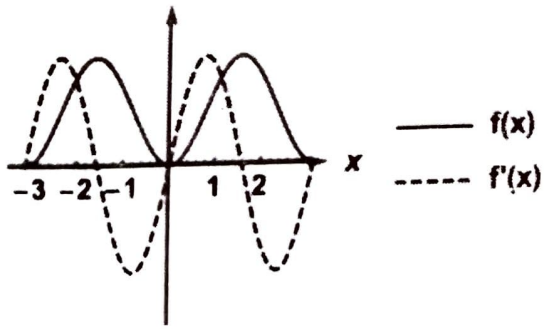
A. $a > 0$

B. $a < 0$

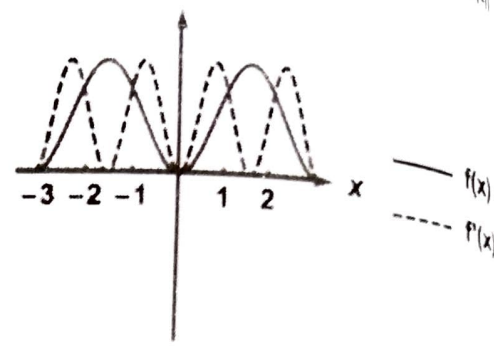
C. $a = \frac{1}{2}$

D. sign of a cannot be determined

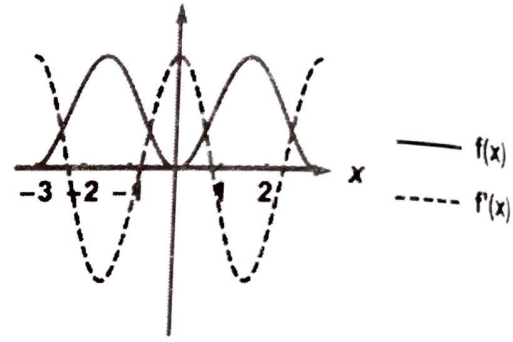
10. The following graphs represent the function $f(x)$ (in bold line) and its derivative $f'(x)$ (in dotted lines). The graph that shows the correct relation is:



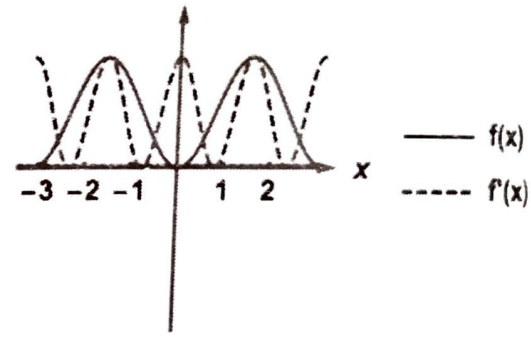
Graph I



Graph II



Graph III



Graph IV

A. Graph III B. Graph I

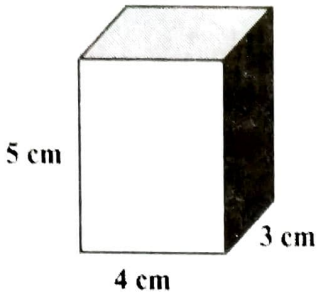
C. Graph II D. Graph IV

11. Two points are taken at random on the given straight line segment of length 'a'. The probability for the distance between them to exceed a given length 'c' is, where $0 < c < a$:

A. $\left(1 - \frac{a^2}{c^2}\right)$ B. $\left(1 - \frac{c^2}{a^2}\right)$

C. $\left(1 - \frac{c}{a}\right)^2$ D. $\left(1 - \frac{a}{c}\right)^2$

12. A block has edge lengths of 3 cm, 4 cm, and 5 cm (as shown), where opposite faces have the same colour. If the block is rolled, which colour is most likely to come up on top? (Assume the block's mass is evenly distributed.)



A. Each colour is equally likely
B. Black
C. Gray
D. White

13. In a computer graphics program, the coordinates of each point of the image given below is transformed using the matrix $\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$ followed by the transformation matrix $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$. The transformed image will be:

IMAGE

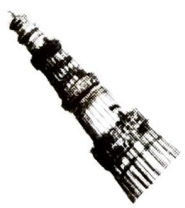


IMAGE I

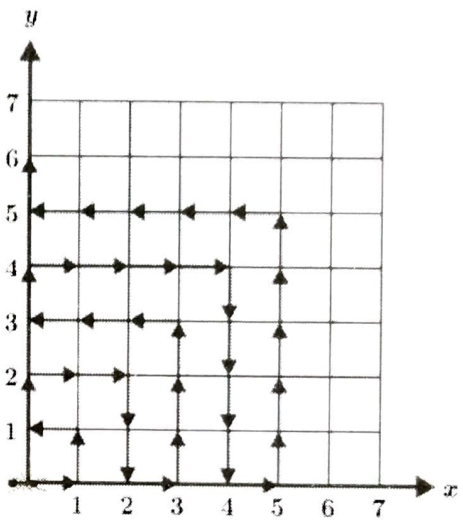
IMAGE II

IMAGE III

IMAGE IV

A. IMAGE II
B. IMAGE I
C. IMAGE IV
D. IMAGE III

14. An ant begins from (0, 0) in the xy-coordinate plane. It then travels to each point with non-negative integer coordinates following the pattern shown below. The ant stops moving after 2018 units, to its final position (x, y). The value of $x + y$ is:



A. 44
B. 50
C. 45
D. 25

15. If $a > 0$ and the equation $|z - a^2| + |z - 2a| = 3$ represents an ellipse, then a lies in the interval:

A. $\left(1, \sqrt{3}\right)$
B. $(1, 3)$
C. $\left(\sqrt{2}, \sqrt{3}\right)$
D. $(0, 3)$

16. Let α_1 and α_2 be roots of the equation $x^2 - x + p = 0$ and let α_3, α_4 be the roots of the equation $x^2 - 4x + q = 0$. If α_1, α_2, α_3, α_4 are in geometric progression. Then the integral values of p and q respectively are:

A. $-6, -32$
B. $-6, 3$
C. $-2, -3$
D. $-2, -32$

17. If $f(x)$ is a polynomial satisfying $\lim\limits_{x\to\infty} \frac{xf(x)}{1+x^5} = 1$, $f(0) = 0$, $f(1) = 2$, $f(2) = 4$, $f(3) = 6$, then $f(4) =$

A. 32
B. 0
C. 8
D. 4

18. Consider the equation

$$|x+1|^{\log_{(x+1)}(3+2x-x^2)} = (x-3)|x|$$

The equation has:

A. unique solution

B. no solution

C. more than two solutions

D. exactly two solutions

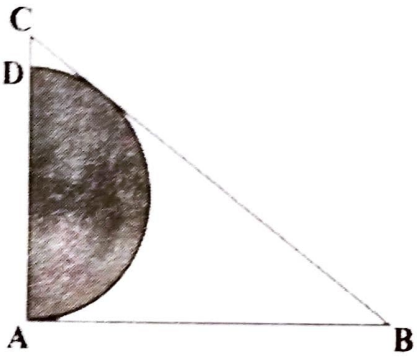
19. The sum of the squares of three distinct real numbers which are in strictly increasing GP is S^2, their sum is αS.

If $S = 10\sqrt{3}$, then the greatest value of the middle term is:

A. 12 B. 5

C. 10 D. 8

20. Given that AB = 12, CD = 1 and the semicircle is tangent to BC, the radius of the semicircle is:



A. 6 B. 9

C. 8 D. 4

21. The number of solutions of $z^3 + \bar{z} = 0$ is:

A. 2 B. 5

C. 3 D. 4

22. Given $f(x) = \begin{cases} 1-|x|, & |x| \le 1 \\ |x|-1 & |x| > 1 \end{cases}$ and $g(x) = f(x-1) + f(x+1)$, the value of the integral

$$\int_{-3}^{5} g(x)dx \text{ is:}$$

A. 2 B. 24

C. 8 D. 40

23. Let $f : [-1, 0] \to \mathbf{R}$ be a function differentiable within the domain such that

$$\int_{-1}^{0} (f(x))^2 dx = 10, \quad f(-1) = 2.$$

Then the value of the integral

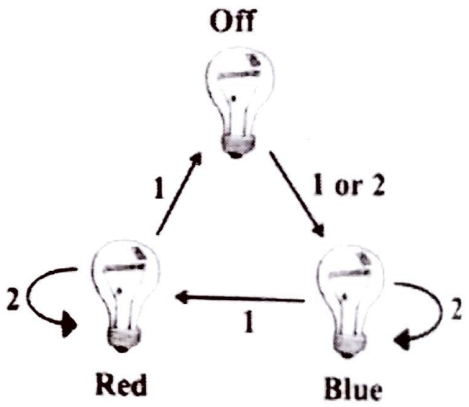
$$\int_{-1}^{0} xf'(x)f(x)dx \text{ is:}$$

A. −3 B. −6

C. −14 D. −7

24. There is a light in another room that can be in one of three states: *blue*, *red* or *off*. In your room, you have two buttons numbered 1 and 2 that control the state of the light. The state of light changes as follows:

- If you press button 1, then the light changes to the next state clockwise around the diagram.
- If you press button 2 while the light is *blue* or *red* the state of the light will not change.
- If you press button 2 while the light is *off* the light will change to *blue*.



You want to make sure that the light is *off* but since the light is in another room, you do not know what state the light is currently in. The shortest sequence of button presses that will guarantee that the light will be off is $x_1x_2 \ldots x_n$ where $x_i = 1$ or 2 for I = 1, 2, ..., n.

The value of n is:

A. 6 B. 7

C. 5 D. 4

25. Let P(x) denotes the probability of the occurrance of event X. Then, all those points $(x, y) = (P(A), P(B))$, in a plane which satisfies the condition $P(A \cup B) \ge \frac{3}{4}$ and $\frac{1}{8} \le P(A \cap B) \le \frac{3}{8}$ implies:

A. $\frac{7}{8} \le x + y \le \frac{11}{8}$ B. $x + y < \frac{7}{8}$

C. $x + y > \frac{11}{8}$ D. $x + y \le \frac{11}{8}$

26. The period of the function which satisfies the relation
$f(x) + f(x + 4) = f(x + 2) + f(x + 6)$ is:
A. 6 B. 5
C. 8 D. 4

27. $\lim_{x \to \infty} (x^2 \ln(\cot^{-1} x)) =$

A. $\frac{2}{3}$ B. $\frac{1}{3}$

C. $-\frac{1}{3}$ D. $-\frac{2}{3}$

28. A maths professor asked one of his students to visit his house regarding some work. The student asked the professor his house number. The professor replied in a strange way:

- If my house number is a multiple of 3, then it is a number from 50 to 59.
- If my house number is not a multiple of 4, then it is a number from 60 to 69.
- If my house number is not a multiple of 6, then it is a number from 70 to 79.

The sum of the prime factors of the professor's house number is:
A. 18 B. 21
C. 26 D. 73

29. The sum of the squares of three distinct real numbers which are in strictly increasing GP is S^2, their sum is αS.
If $r = 2$, then the value of $[\alpha^2]$ is (where [.] denotes the greatest integer function and r is common ratio of GP)
A. 2 B. 0
C. 1 D. 3

30. Let $f : R \to R$ be given by $f(x) = (x + 1)^2 - 1$, $x \ge -1$. Then $f^{-1}(x)$ is:
A. Does not exist, as f is not onto
B. $-1 - \sqrt{x+1}$
C. Does not exist, as f is not one-one
D. $-1 + \sqrt{x+1}$

31. Let α, β, γ be three numbers such that $\alpha^2 + \beta^2 + \gamma^2 - \gamma = 0$ and $z = \frac{\alpha + i\beta}{1 - \gamma}$.
Value of α is:

A. $\frac{z - \bar{z}}{2(1 + |z|^2)}$ B. $\frac{\bar{z}}{2(1 + |z|^2)}$

C. $\frac{z}{2(1 + |z|^2)}$ D. $\frac{z + \bar{z}}{2(1 + |z|^2)}$

32. The real values of x that satisfy the equation $x^2 - 2^x = 0$ is:
A. 2 B. 1
C. 3 D. 4

33. The number of integral solutions of the equation $x + y + z = 0$, where x, y, z are all ≥ -5 is:
A. 455 B. 105
C. 135 D. 136

34. If $\sum_{n=1}^{k} \left[\frac{1}{3} + \frac{n}{90}\right] = 21$, where $[x]$ denotes the integral part of x, then k equal to:
A. 80 B. 85
C. 84 D. 83

35. The diagram below shows a grid of 2×4 dots (of negligible size) that alternate between black and white. The number of ways there are to draw ten line segments between the dots such that

- each white dot is connected to exactly one black dot (and vice versa),
- no segment passes through more than two dots, and
- none of the segments intersect each other

is:

A. 11 B. 12
C. 10 D. 17

Directions (Q. No. 36 and 37): *Read the following information and solve the questions that follow:*

Each of the digits 1, 2, ..., 8 are placed exactly once in the grid below so that the equations are true.

A	+	B	=	C
+		×		
D	÷	E	=	F
=		=		
G		H		

36. $A \times B \times C \times E =$

A. 240 B. 70
C. 48 D. 40

37. $A + B + D + E =$

A. 15 B. 18
C. 13 D. 16

38. If $\vec{a}, \vec{b}$ and $\vec{c}$ are non-coplanar vectors and λ is a real number, then

$\lambda\left(\vec{a}+\vec{b}\right).\left(\lambda^2\vec{b}\times\lambda\vec{c}\right) = \vec{a}.\left(\left(\vec{b}+\vec{c}\right)\times\vec{b}\right)$ for

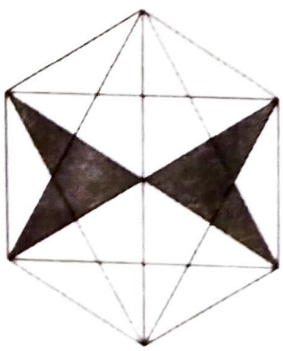
A. exactly two values of λ.
B. exactly one value of λ.
C. no value of λ.
D. exactly three values of λ.

39. Let $P(x) = (x-1)(x-2)(x-3)...(x-50)$ and $Q(x) = (x+1)(x+2)(x+3)...(x+50)$.

If $P(x)\,Q(x) = a_{100}x^{100} + a_{99}x^{99} + a_{98}x^{98} + ... + a_1x^1 + a_0$, then the value of $a_{100} - a_{99} - a_{98} - a_{97}$ is:

A. 21926 B. 42926
C. 0 D. −1

40. The area of a regular hexagon is H and the area of the shaded region in it is S. The ratio of H : S is:



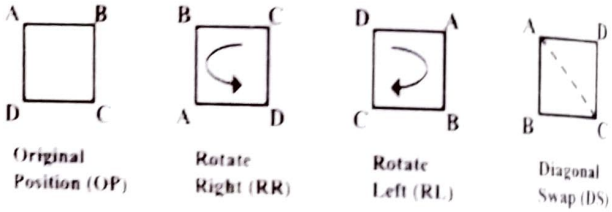
A. 7 : 2 B. 6 : 1
C. 5 : 1 D. 9 : 2

41. Let A be the area bounded by the curve $y = -k^{17}x^{16} + k$ and the x axis. The value of 68A is:

A. 128 B. 68
C. 34 D. 32

42. A square dance has the following commands:

- "Rotate Right" means everyone moves one pace around the square, counterclockwise.
- "Rotate Left" means everyone moves one pace around the square, clockwise.
- "Diagonal Swap" means the person in the lower-left position and the person in the upper-right position switch places, while the other two people stay still.



A choreographer choreographs a sequence that is made of the steps, starting from the original position

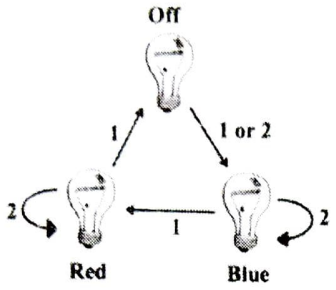
RR → RR → DS → RL → DS

The position of A after 21 repetitions of the above steps is:

A. The upper right corner
B. The upper left corner
C. The lower right corner
D. The lower left corner

43. There is a light in another room that can be in one of three states: *blue*, *red* or *off*. In your room, you have two buttons numbered 1 and 2 that control the state of the light. The state of light changes as follows:

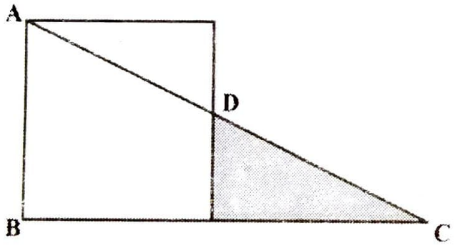
- If you press button 1, then the light changes to the next state clockwise around the diagram.
- If you press button 2 while the light is *blue* or *red* the state of the light will not change.
- If you press button 2 while the light is *off* the light will change to *blue*.



You want to make sure that the light is *off* but since the light is in another room, you do not know what state the light is currently in. The shortest sequence of button presses that will guarantee that the light will be off is $x_1x_2 \ldots x_n$ where $x_i = 1$ or 2 for I = 1, 2, ..., n. $x_1 + x_2 + \ldots + x_n =$

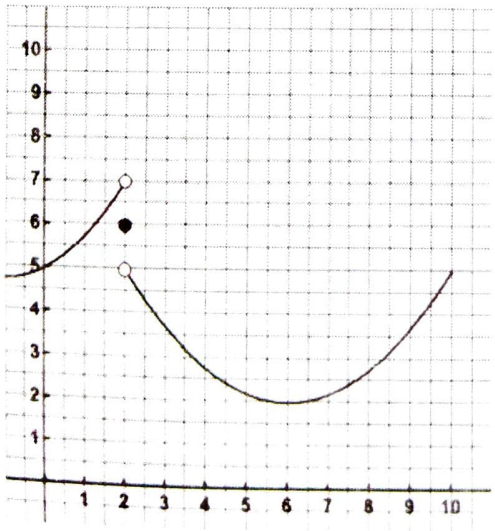
A. 6 B. 9
C. 7 D. 8

44. A right triangle ABC with side AB = x and BC = 12 intersects a square with side AB at the point D. The value of x that maximizes the area of the shaded portion of the triangle is:



A. 6 B. 12
C. 3 D. 4

45. The graph of a function $f(x)$ is given below. The value of $\lim_{x \to 6} f(f(x))$ is:



A. 6 B. 7
C. does not exist D. 5

46. If the root of the equation $x^2 + 2ax + b = 0$ are real and distinct and they differ by at most $2m$, then b lies in the interval:

A. $(a^2 + m^2, a^2)$ B. $[a^2 - m^2, a^2)$
C. $(a^2, a^2 + m^2)$ D. $[a^2, a^2 - m^2]$

47. Let $F(x)$ be a function defined by $F(x) = x - [x], 0 \neq x \in R$, where $[x]$ is the greatest integer less than or equal to x. Then the number of solutions of $F(x) + F(1/x) = 1$ is/are:

A. 2 B. 0
C. 1 D. infinite

48. If $f(x) = \max\left(\left|2\sin y - x\right|\right)$ for $y \in (-\infty, \infty)$, then the minimum value of $f(x)$ is:

A. 2 B. 1
C. 0 D. –2

49. The distance between the two foci F and F′ of an ellipse is 8 units. P is any point on the ellipse such that $|PF| + |PF'| = 32$. The maximum area of the triangle PFF′ is:

A. $4\sqrt{238}$ B. $4\sqrt{240}$
C. $3\sqrt{240}$ D. $3\sqrt{238}$

50. The determinant $\begin{vmatrix} a & b & a\alpha - b \\ b & c & b\alpha - c \\ 2 & 1 & 0 \end{vmatrix} = 0$, $\alpha \neq 0$ if:

A. a, b, c are in HP
B. a, b, c are in AGP
C. a, b, c are in AP
D. a, b, c are in GP

51. Let α, β, γ be three numbers such that $\alpha^2 + \beta^2 + \gamma^2 - \gamma = 0$ and $z = \dfrac{\alpha + i\beta}{1 - \gamma}$.

Value of β is:

A. $\dfrac{z - \bar{z}}{2(1 + |z|^2)}$ B. $\dfrac{z}{2(1 + |z|^2)}$

C. $\dfrac{(\bar{z} - z)i}{2(1 + |z|^2)}$ D. $\dfrac{2z}{1 + |z|^2}$

52. Let P(t) represent the amount of chemical a factory produces as a function of time t (in hours). The rate of change of chemical production satisfies the differential equation

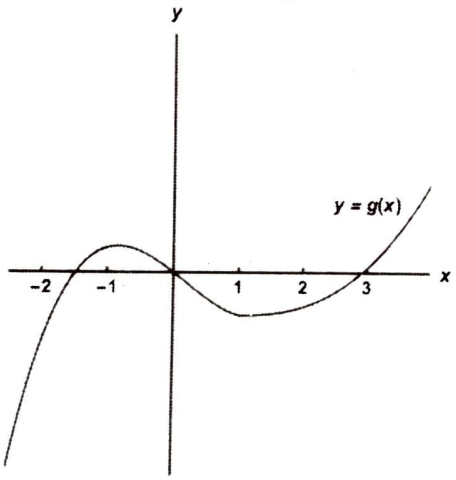
$$\frac{dP(t)}{dt} = -\ln 3\ P(t)\left(1-\frac{P(t)}{3}\right)$$

If the factory alarm is raised when chemical production exceeds 4 in 4 hours, the inequality that represents the maximum initial amount of chemical that guarantees the alarm will not be raised is:

A. $P(0) < \frac{108}{37}$ B. $P(0) < \frac{108}{373}$

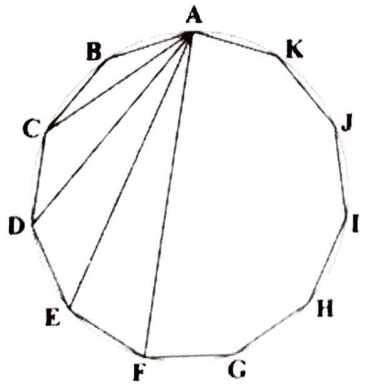
C. $P(0) < \frac{972}{337}$ D. $P(0) < \frac{972}{323}$

53. The graph represents the function $y = g(x)$. Which of the following statements is true?



A. $g'(-2) > g'(2) > g'(0)$
B. $g'(-2) > g'(0) > g'(2)$
C. $g'(0) > g'(2) > g'(-2)$
D. $g'(2) > g'(-2) > g'(0)$

54. A regular 11-sided polygon ABCDEFGHIJK is inscribed in a unit circle, as shown below.



The value of AB × AC × AD × AE × AF is:

A. 11 B. 22

C. $\sqrt{11}$ D. $\sqrt{22}$

55. $\vec{a}, \vec{b}, \vec{c}$ are three vectors of magnitude $\sqrt{2}$, 1 and $\frac{1}{\sqrt{2}}$ respectively, and satisfying $\left|\left[\vec{a}, \vec{b}, \vec{c}\right]\right| = 1$, then the value of $\left(\vec{a}+2\vec{b}+\vec{c}\right).\left(\left(\vec{a}\times\vec{b}\right)\times\left(\vec{a}-\vec{b}\right)+\vec{c}\right) =$

A. 0 B. 5

C. 6.5 D. 8

56. The range of the function f defined by $f(x) = \left[\frac{1}{\sin\{x\}}\right]$ where $[x]$ & $\{x\}$ respectively denote the greatest integer and fractional part of x, is:

A. I, the set of integers
B. N, the set of natural numbers
C. W, the set of whole numbers
D. {2, 3, 4, ...}

57. The sum of the squares of three distinct real numbers which are in strictly increasing GP is S^2, their sum is αS.
Maximum interval for α^2 is:

A. $\left(\frac{1}{3}, 3\right)$ B. (1, 2)

C. $\left(\frac{1}{3}, 1\right) \cup (1, 3)$ D. $\left(\frac{1}{3}, 1\right)$

58. If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors and $\vec{a} = \alpha\left(\vec{a}\times\vec{b}\right)+\beta\left(\vec{b}\times\vec{c}\right)+\gamma\left(\vec{c}\times\vec{a}\right)$ and $\left[\vec{a}, \vec{b}, \vec{c}\right] = 1$, then $\alpha + \beta + \gamma =$

A. $-|\vec{a}|^2$ B. $|2\vec{a}|^2$

C. $|\vec{a}|^2$ D. 0

59. If $\vec{a}, \vec{b}$ and $\vec{c}$ are non-coplanar vectors, then $\left[\left(\vec{a}+\vec{b}\right)\times\left(\vec{b}-\vec{c}\right)\ \left(\vec{b}+\vec{c}\right)\times\left(\vec{c}+\vec{a}\right)\ \left(\vec{c}-\vec{a}\right)\times\left(\vec{a}+\vec{b}\right)\right] =$

A. $2\left[\left[\vec{a}\ \vec{b}\ \vec{c}\right]\right]^2$ B. $3\left[\left[\vec{a}\ \vec{b}\ \vec{c}\right]\right]^2$

C. $\left[\left[\vec{a}\ \vec{b}\ \vec{c}\right]\right]^2$ D. $4\left[\left[\vec{a}\ \vec{b}\ \vec{c}\right]\right]^2$

60. Consider the system of linear equations $2x + 3y + 5z = 9$, $7x + 3y - 2z = 8$ and $2x + 3y + \lambda z = \mu$.

Under what condition does the above system of equations have infinitely many solutions?

A. $\lambda = 9$ and $\mu = 5$
B. $\lambda = 9$ and $\mu \neq 5$
C. $\lambda = 5$ and $\mu = 9$
D. $\lambda = 5$ and $\mu \neq 9$

61. Consider the system of linear equations

$2x + 3y + 5z = 9$, $7x + 3y - 2z = 8$ and $2x + 3y + \lambda z = \mu$.

Under what condition does the above system of equations have a unique solution?

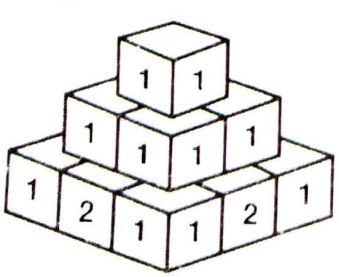
A. λ has any real value and $\mu \neq 9$
B. $\lambda \neq 5$ and $\mu = 7$ only
C. $\lambda = 5$ and $\mu = 9$
D. $\lambda \neq 5$ and μ has any real value

62. Let $f : [0, 1] \to R$ be a continuous function such that $f(x)\, f(x - 1) = 1$. Then the value of the integral $\int_0^1 \frac{1}{1+f(x)}dx$ is:

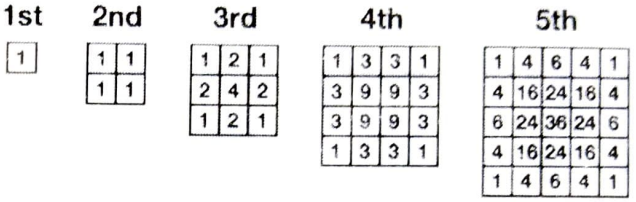
A. 2 B. 0
C. 0.5 D. 1

Directions (Q. No. 63-66): *Read the following information and solve the questions that follow:*

A pyramid is made out of blocks, where the first layer has 1 block, the second layer is a square made up of 4 blocks and the third layer is a square made up of 9 blocks, and so on, so that each n^{th} layer is a square made from n^2 blocks.



Each block has a number written on it. The top block has the number 1 written on it, but every other block has the sum of all the blocks that touch its top side written on it. The first five layers are as follows:



63. The number on the block that is in the 3rd column and the 4th row of the 100th layer is:

A. 590438611224 B. 634062082500
C. 66701250 D. 760874499

64. The sum of the numbers on the outermost blocks of the 10th layer is:

A. 1016 B. 1020
C. 512 D. 508

65. The sum of the numbers on the diagonal blocks in the 9th layer is:

A. 20840 B. 97240
C. 25740 D. 81364

66. The sum of the numbers on the diagonal blocks in the 10th layer is:

A. 306008 B. 369260
C. 369512 D. 97240

67. If λ be an integer and α, β be the roots of $4x^2 - 16x + \lambda = 0$ such that $1 < \alpha < 2$ and $2 < \beta < 3$, then possible values of λ are:

A. {62, 65, 68, 71, 75}
B. {60, 64, 68}
C. {61, 62, 63}
D. {49, 50, ..., 62, 63}

Directions (Q. No. 68-72): *Read the following information and solve the questions that follow:*

Shukla, Mishra, Singh, Kulkarni, Rao, Joshi and Nair are to conduct interviews simultaneously either alone or in pairs at four different locations—Surat, Chandigarh, Delhi and Lucknow. If they travel in pair, they take the same mode of travel. Only one travels by rail, two are travelling by car and the rest travel by air.

- Shukla is going to Lucknow but neither by car nor by air.

- Mishra prefers to travel by car.
- Neither Joshi nor Nair is going to Delhi.
- Only those going to Surat travel by road.
- Kulkarni will assist his friend Mishra.
- The two managers who go to Delhi travel by air.

68. In which of the following pair, both of the managers have same mode of travel?

A. Shukla-Mishra B. Rao-Mishra
C. Nair-Rao D. Kulkarni-Joshi

69. Which of the following is true?

A. Joshi travels by road
B. Nair will assist Rao
C. Shukla conducts interviews alone
D. Kulkarni travels by air

70. Who will conduct interviews at Chandigarh?

A. Mishra B. Nair
C. Singh D. Rao

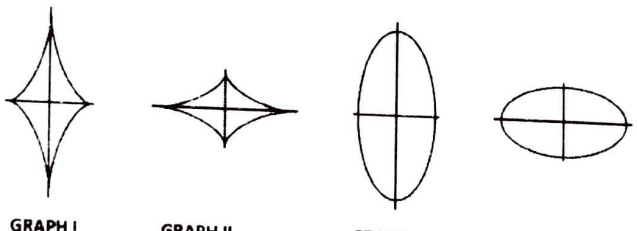
71. Who goes to Delhi?

A. Data inadequate B. Mishra-Kulkarni
C. Kulkarni-Joshi D. Rao-Singh

72. Where will Kulkarni conduct the interviews?

A. Chandigarh
B. Surat
C. Cannot be determined
D. Lucknow

73. The curve defined by the parametric equation $x = 3\cos t$, $y = 4\sin t$ is:



A. GRAPH III B. GRAPH I
C. GRAPH II D. GRAPH IV

74. Which of the following is a function?
(**Note:** [.] & {.} denote the greatest integer and fractional part function respectively)

A. $x!\,\{x\}$ B. $\dfrac{\ln(x-1)}{\sqrt{1-x^2}}$

C. $\dfrac{x!}{\{x\}}$ D. $\dfrac{1}{\ln[1-|x|]}$

75. Let $f(x) = \begin{vmatrix} 1+\sin^2 x & \cos^2 x & 4\sin 2x \\ \sin^2 x & 1+\cos^2 x & 4\sin 2x \\ \sin^2 x & \cos^2 x & 1+4\sin 2x \end{vmatrix}$.

$f(a)$ and $f(b)$ be the least and greatest values of $f(x)$, then

A. $f(a) = 2, f(b) = 6$ and period of $f(x)$ is π
B. $f(a) = -2, f(b) = 6$ and period of $f(x)$ is π
C. $f(a) = 2, f(b) = -6$ and period of $f(x)$ is $\dfrac{\pi}{2}$
D. $f(a) = -2, f(b) = 6$ and period of $f(x)$ is $\dfrac{\pi}{2}$

76. The solution of $x - 1 = (x - [x])(x - \{x\})$ (where $[x]$ and $\{x\}$ are the integral and fractional part of x) is:

A. $x \in R \sim [1, 2]$ B. $x \in R \sim [1, 2)$
C. $x \in R \sim (1, 2)$ D. $x \in [1, 2)$

77. The total number of solutions of $2^{\cos x} = |\sin x|$ in $\{-2\pi, 5\pi]$ is:

A. 15 B. 12
C. 14 D. 16

78. Let $w \neq 1$ is nth root of unity, then value of $\sum_{k=0}^{n-1} \left|z_1 + w^k z_2\right|^2$ is:

A. $|z_1|^2 + |z_2|^2$ B. $n\left(|z_1|^2 + |z_2|^2\right)$
C. $n\left(|z_1| + |z_2|\right)^2$ D. $\left(|z_1| + |z_2|\right)^2$

79. If a function $f(x)$ satisfies the functional equation $f(\tan\theta) = \sin^2 2\theta$ for all real θ, then $\int_0^1 f(x)dx =$

A. $\pi - 2$ B. $\dfrac{\pi}{4}$
C. $\dfrac{\pi}{2}$ D. $\dfrac{\pi}{2} - 1$

80. The curve $x^3 + y^3 + x^2y + 7 = 0$ intersects the line $9x + y = 0$ at the point (a, b). The slope of the tangent to the curve $x^3 + y^3 + x^2y + 7 = 0$ at the point (a, b) is:

A. $\frac{23}{244}$ B. $\frac{13}{244}$

C. $\frac{21}{244}$ D. $\frac{15}{244}$

81. The sum of the squares of three distinct real numbers which are in strictly increasing GP is S^2, their sum is αS.

If $\alpha^2 = 2$, then the value of $[r]$ is (where [.] denotes the greatest integer function and r is common ratio of GP)

A. 2 B. 0

C. 1 D. 3

82. Let $f(x) = 6x^2 + 7x - 5$. The average rate of change of $f(x)$ when x changes from 0 to 18 is the same as the rate of change of $f(x)$ at $x = a$. The value of $a^3 + a^2 + 90$ is:

A. 666 B. 900

C. 810 D. 342

83. A straight line in the xy-plane passes through the point (3, 48) in the first quadrant and intersects the positive x-axis and y-axis at points P and Q respectively. If O is the origin, the minimum value of OP × OQ is:

A. 144 B. 72

C. 576 D. 288

84. If $x^a = y^b = z^c$, where a, b, c are unequal positive numbers and x, y, z are in GP, then $a^3 + c^3$ is:

A. $> 2b^3$ B. $\geq 3b^3$

C. $< 2b^3$ D. $\leq 2b^3$

85. Period of $f(x) = x - [x + a] - b$, where $a, b \in R^+$ and, [.] denotes the greatest integer function, is:

A. 1 B. b

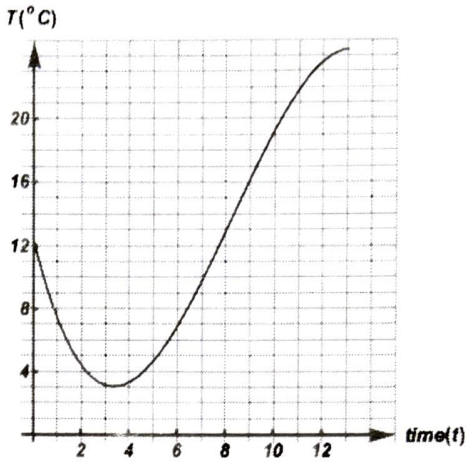
C. $|a - b|$ D. a

86. If ab^2c^3, $a^2b^3c^4$, $a^3b^4c^5$ are in AP ($a, b, c > 0$), then the minimum value of $a + b + c$ is:

A. 1 B. 5

C. 3 D. 9

87. The maximum temperature (T) of a city X for the fist two weeks of January (1st-14th) is plotted as a function of time t.



The approximate rate of change of temperature on 9th January is:

A. 3.5 B. 1

C. −3 D. 2

88. A game is played with the following rules:

- Player X starts with 1 chip
- In each round of play, a fair six-sided die is rolled. If the result is 1, 2, or 3, the player loses a chip. If the result is 4 or 5, the player gains a chip. If the result is 6, 2 chips are gained.
- If at any time the player X has no chips, the game is lost.
- If at any time the player has 4 or more chips, he wins the game.

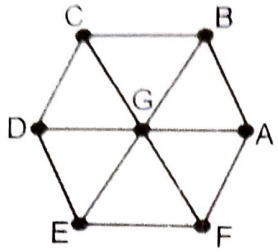
What is the probability that the player will win the game?

A. $\frac{7}{36}$ B. $\frac{19}{108}$

C. $\frac{1}{3}$ D. $\frac{4}{27}$

Directions (Q. No. 89 and 90): *Read the following information and solve the questions that follow:*

A bug stands on a random point of the lattice below. Each point is equally likely to be the starting point. Every minute, the bug selects an adjacent point at random and moves to it. Each point is equally likely to be the bug's starting point and each adjacent point is equally likely to be chosen. Also, assume starting at A will "reach" the point A in 0 moves.



89. If the bug starts from point G, the probability of reaching A in 2 moves or less is:

A. 1 B. $\frac{5}{9}$

C. $\frac{1}{3}$ D. $\frac{5}{18}$

90. Starting from any point, what is the probability that the bug reaches point A in 2 moves or less?

A. $\frac{5}{18}$ B. $\frac{1}{6}$

C. $\frac{22}{63}$ D. $\frac{1}{18}$

91. A and B are points in the first quadrant such that the slopes of the line segments OA and OB are 3 and 1/3, respectively, where O is the origin. If |OA| = |OB| then the equation of line which is parallel to the line segment AB and passes through the point (–3, 7) is:

A. $y = 2x + 13$ B. $y = -2x + 1$

C. $y = -x + 4$ D. $y = x + 10$

92. Let $f(x) = 3x^{10} - 7x^8 + 5x^6 - 21x^3 + 3x^2 - 7$.

The value of $\lim_{h\to 0} \frac{f(1-h)-f(1)}{h^3+3h}$ is:

A. $\frac{53}{3}$ B. 13

C. $\frac{22}{3}$ D. $\frac{13}{3}$

Directions (Q. No. 93 and 94): *Read the following information and solve the questions that follow:*

P = Q means Q is the father of P

P * Q means P is the sister of Q

P ? Q means Q is the mother of P

P $ Q means P is the brother of Q

P @ Q means Q is son of P

P # Q means P is the daughter of Q

93. Which of the following is correct?

A. V # T * P means P is maternal uncle of V

B. L @ M $ R means R is paternal uncle of L

C. D ? V # T means D is grand-daughter of T

D. M @ R $ D ? V means M and D are husband and wife

94. Which of the following indicates A is grandfather of B?

A. B # L # A

B. M # A = N = B

C. L * B = S $ Q = A

D. B $ L # Q # A

95. The sum of the squares of three distinct real numbers which are in strictly increasing GP is S^2, their sum is αS.

If we drop the condition that the GP is strictly increasing and take $r^2 = 1$, (where r is common ratio of GP) then the value of α can be

A. 0 B. $\pm\frac{1}{\sqrt{3}}$

C. ± 1 D. $\pm\frac{1}{\sqrt{2}}$

96. If β is the root of the polynomial $x^2 + 24x + 1$, then $\lim_{x\to\beta} \frac{1-\cos(x^2+24x+1)}{(x-\beta)^2}$ is:

A. 144 B. 286

C. 12 D. 576

97. A function $f(x)$ is defined by $f(x) = \frac{1}{2}\left\{f(xy)+f\left(\frac{x}{y}\right)\right\}$ for all $x, y \in R^+$, the set of positive real numbers. If $f(1) = 0, f'(5) = 5$, then $f''(5) =$

A. –1 B. 1

C. –5 D. 5

98. If $f(x)$ is defined in $(0, 1)$, then the domain of definition of $f(\sin x)$ is:

A. $(2n\pi, (2n + 1)\pi), n \in Z$

B. $((n - 1)\pi, (n + 1)\pi), n \in Z$

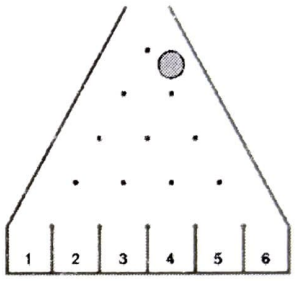
C. $\left((2n-1)\frac{\pi}{2}, (2n+3)\frac{\pi}{2}\right), n \in Z$

D. $\left((2n+1)\frac{\pi}{2}, (2n+3)\frac{\pi}{2}\right), n \in Z$

99. A batsman in his 16th innings makes a score of 70 runs and thereby increases his average by 2 runs. If he has never been not out, then his average after 16th inning is:

A. 38 B. 36
C. 42 D. 40

100. The figure shows a peg board used for a game show. Whenever the gray chip falls onto a peg, it has an equal chance to bounce either to the left or the right peg right.



The probability of the chip falling in slot 2 is:

A. $\frac{1}{16}$ B. $\frac{1}{8}$

C. $\frac{1}{4}$ D. $\frac{3}{8}$

ANSWERS

1	2	3	4	5	6	7	8	9	10
D	A	C	D	D	D	D	B	A	B
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
C	D	A	B	D	D	A	B	C	D
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
B	B	A	A	A	C	C	B	A	D
31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
D	A	D	A	A	D	C	C	B	D
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
A	C	D	D	D	B	D	A	B	D
51	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
C	D	A	C	C	B	C	C	D	C
61	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**
D	C	D	D	A	D	D	C	C	B
71	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**
D	B	A	A	B	D	C	B	D	D
81	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**	**90**
A	B	C	A	A	C	A	C	D	C
91	**92**	**93**	**94**	**95**	**96**	**97**	**98**	**99**	**100**
C	A	B	C	B	B	A	A	D	A

EXPLANATORY ANSWERS

1. The maximum marks that can be assigned to any question is 16. If x_i are the marks assigned to ith question. Then

$x_1 + x_2 + x_3 + \ldots + x_8 = 30$ and $2 \le x_i \le 16$

For $i = 1, 2, 3, \ldots 8$

Put $x_i = y_i + 2$, so that $y_i \ge 0$ and

$y_1 + y_2 + \ldots + y_8 = 14$

The number of non-negative integral solution of this equation is

$= {}^{14+8-1}C_{8-1}$

$= {}^{21}C_7$.

4. Area of the circle $= \pi a^2$

Area of the ellipse $= \pi ab$

Hence we have $\dfrac{\pi ab - \pi a^2}{\pi ab} = \left(1 - \dfrac{a}{b}\right)$

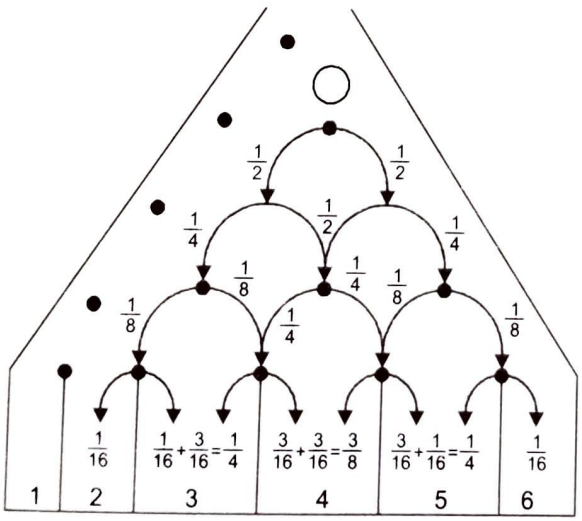
Thus $\quad P = \left(1 - \dfrac{a}{b}\right)$

$\Rightarrow \quad \dfrac{a}{b} = 1 - P$

Eccentricity of an ellipse

$= \sqrt{1 - \left(\dfrac{a}{b}\right)^2} = \sqrt{1 - (1-P)^2}$.

5.



Probability of chip falling:

$2 \rightarrow \dfrac{1}{16}$

$3 \rightarrow \dfrac{1}{4}$

$4 \rightarrow \dfrac{3}{8}$

$5 \rightarrow \dfrac{1}{4}$

$6 \rightarrow \dfrac{1}{16}$

Highest probability of slot 4 so ball would most likely fall to slot 4.

15. The equation $|z - a^2| + |z - 2a| = 3$ will represent an ellipse if

$\Rightarrow |a^2 - 2a| < 3$

$\Rightarrow -3 < a^2 - 2a < 3$

$\Rightarrow -2 < (a-1)^2 < 4$

$\Rightarrow (a-1)^2 < 4$

$\Rightarrow -2 < (a-1) < 2$

$\Rightarrow -1 < a < 3$

$\Rightarrow a \in (0, 3)$ $\quad [\because a > 0]$

21. We have, $z^3 + \bar{z} = 0$

Let $z = x + iy$

Then,

$(x + iy)^3 + (x - iy) = 0$

$x^3 + i^3y^3 + 3(x)(iy)(x + iy) + x - iy = 0$

$x^3 + iy^3 + 3x^2iy + 3xi^2y^2 + x - iy = 0$

$x^3 + iy^3 + 3x^2iy - 3xy^2 + x - iy = 0$

$(x^3 - 3xy^2 + x) + (y^3 + 3x^2y - y)i = 0$

$x^3 - 3xy^2 + x = 0$

$y^3 + 3x^2y - y = 0$

$x(x^2 - 3y^2 + 1) = 0$

$\Rightarrow \quad x = 0$

or $\quad x^2 - 3y^2 + 1 = 0$

$y(y^2 + 3x^2 - 1) = 0$

$\Rightarrow \quad y = 0$

or $y^2 + 3x^2 - 1 = 0$

If $x = 0$,

$$y^2 + 0 - 1 = 0$$

$$y = \pm 1$$

If $y = 0$,

$$x^2 - 3 \times 0 + 1 = 0$$

$$x^2 = -1$$

$$x^2 = i^2$$

$$x = \pm i$$

So there are five solutions of the given.

26. We have

$f(x) + f(x + 4) = f(x + 2) + f(x + 6)$ ...(*i*)

Let replace x by $x + 2$

$f(x + 2) + f(x + 6) = f(x + 4) + f(x + 8)$ ...(*ii*)

From eqns (*i*) and (*ii*) we get

$f(x) + f(x + 4) = f(x + 4) + f(x + 8)$

or $f(x) = f(x + 8)$

So the period is '8'.

29. Let three number of G.P. are $\frac{a}{r}$, a, ar

Then, $\frac{a^2}{r^2} + a^2 + a^2r^2 = S^2$ $[\because r = 2]$

$$\frac{a^2}{4} + a^2 + 4a^2 = S^2$$

$$\frac{21a^2}{4} = S^2 \quad ...(i)$$

$$\frac{a}{r} + a + ar = \alpha S \text{ [square both sides]}$$

$$\frac{a^2}{r^2} + a^2 + a^2r^2 + 2\left[\frac{a^2}{r} + a^2r + a^2\right] = \alpha^2 S^2$$

$$S^2 + 2a^2\left[\frac{1}{r} + r + 1\right] = \alpha^2 S^2$$

Put $S^2 = \frac{21a^2}{4}$ and $r = 2$

$$\frac{21a^2}{4} + 2a^2\left[\frac{1}{2} + 2 + 1\right] = \alpha^2 \times \frac{21a^2}{4}$$

$$\frac{21}{4} + 2 \times \frac{7}{2} = \alpha^2 \times \frac{21}{4}$$

$$\frac{21 + 28}{4} = \alpha^2 \times \frac{21}{4}$$

$$\frac{49}{4} = \alpha^2 \times \frac{21}{4}$$

$$\alpha^2 = \frac{7}{3}$$

$$[\alpha^2] = \left[\frac{7}{3}\right]$$

$$= [2.33]$$

$$= 2.$$

30. $f(x) = (x + 1)^2 - 1$, $x \geq -1$

Let

$$y = (x + 1)^2 - 1$$

$$y + 1 = (x + 1)^2$$

$$(x + 1) = \sqrt{y+1}$$

$$x = -1 + \sqrt{y+1}$$

$\therefore$ $f^{-1}(x) = -1 + \sqrt{x+1}$.

31. We have $z = \frac{\alpha + i\beta}{1 - \gamma}$

$$z = \frac{\alpha}{1-\gamma} + i\frac{\beta}{1-\gamma}$$

$$\bar{z} = \frac{\alpha}{1-\gamma} - i\frac{\beta}{1-\gamma}$$

$$\frac{z + \bar{z}}{2} = \frac{\alpha}{1-\gamma} \quad ...(i)$$

$$|z|^2 = \frac{\alpha^2}{(1-\gamma)^2} + \frac{\beta^2}{(1-\gamma)^2}$$

$$= \frac{\alpha^2 + \beta^2}{(1-\gamma)^2}$$

$$|z|^2 = \frac{\gamma(1-\gamma)}{(1-\gamma)^2} \qquad [\because \alpha^2 + \beta^2 = \gamma(1-\gamma)$$

$$|z|^2 = \frac{\gamma}{1-\gamma}$$

$$\gamma = \frac{|z|^2}{1+|z|^2}$$

Put in eq (*i*) we get,

$$\frac{\alpha}{1-\frac{|z|^2}{1+|z|^2}} = \frac{z+\bar{z}}{2}$$

$$\frac{\alpha(1+|z|^2)}{1} = \frac{z+\bar{z}}{2}$$

$$\alpha = \frac{z+\bar{z}}{2(1+|z|^2)}.$$

32. $x^2 - 2^x = 0$

$x^2 = 2^x$

This is satisfy at $x = 2$.

33. We have

$x \geq -5$, $y \geq -5$ and $z \geq -5$

where x, y, z are integers

Let $u = x + 5$

$v = y + 5$

$w = z + 5$

Then,

$x \geq -5, \Rightarrow u \geq 0$

$y \geq -5, \Rightarrow v \geq 0$

$z \geq -5, \Rightarrow w \geq 0$

Thus we have,

$(u - 5) + (v - 5) + (w - 5) = 0$

$u + v + w = 15$

where $u \geq 0$, $v \geq 0$ and $w \geq 0$

Total number of solution of this equation is

$$^{15+3-1}C_{3-1} = {}^{17}C_2$$

$$= \frac{17 \times 16}{2 \times 1}$$

$$= 136.$$

For Q.No. 36 and 37

A = 1	+	B = 4	=	C = 5
+		×		
D = 6	:	E = 2	=	F = 3
=		=		
G = 7		H = 8		

36. A × B × C × E

= 1 × 4 × 5 × 2

= 40.

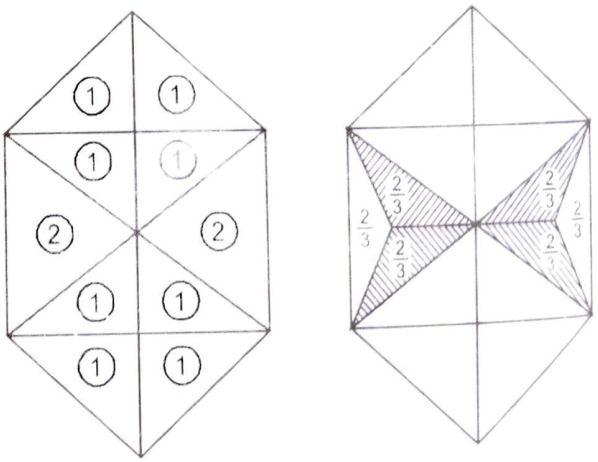
37. A + B + D + E

= 1 + 4 + 6 + 2

= 13.

40. ∵ Figure is symmetrical.

Let the area of Hexagon is 12 unit^2

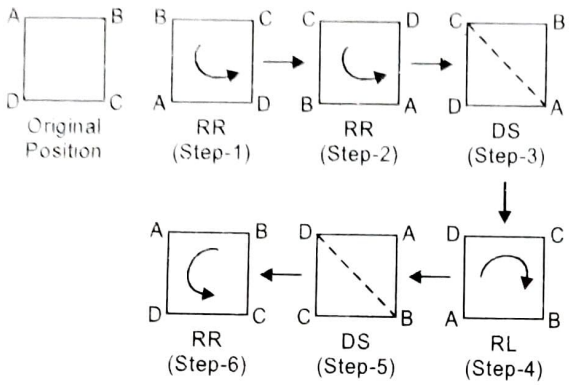


Area of shaded region

$$= \frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3} = \frac{8}{3} \text{ unit}^2$$

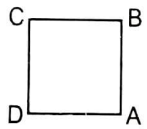
Required ratio H : S $= \dfrac{12}{\left(\frac{8}{3}\right)} = \dfrac{9}{2}$.

42. RR → RR → DS → RL → DS



Square A reached his original position after 6-step, after 21 repetition step will be 3rd step (6 × 3 + 3)

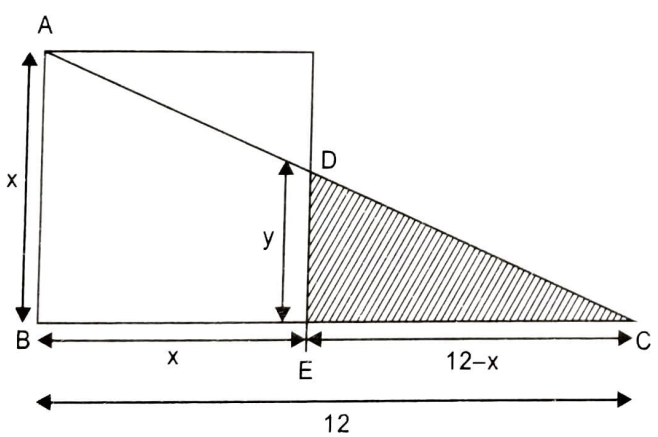
3rd step



The lower right corner.

44. Let DE = y

ΔABC and ΔDEC are similar triangles



Then, $\frac{x}{y} = \frac{12}{12-x}$

$$12y = 12x - x^2$$

$$y = x - \frac{x^2}{12}$$

Shaded region area

$$\Delta = \frac{1}{2} \times (12-x)\left(x - \frac{x^2}{12}\right)$$

$$= \frac{(12-x)(12x-x^2)}{24}$$

$$= \frac{1}{24}\left[x(12-x)^2\right]$$

$$= \frac{1}{24}\left[x(144 + x^2 - 24x)\right]$$

$$= \frac{1}{24}\left[144x + x^3 - 24x^2\right]$$

$$= 6x + \frac{x^3}{24} - x^2$$

For maxima and minima

$$\frac{d\Delta}{dx} = 0$$

$$6 + \frac{3x^2}{24} - 2x = 0$$

$$x^2 + 48 - 16x = 0$$

$$x^2 - 12x - 4x + 48 = 0$$

$$x(x-12) - 4(x-12) = 0$$

$$(x-4)(x-12) = 0$$

$x = 4$, $x = 12$ (x value will not be 12)

So, $x = 4$.

46. $x^2 + 2ax + b = 0$

We know that

Roots real and distinct iff $D > 0$

$$(2a)^2 > 4b > 0$$

$$4a^2 > 4b$$

$$a^2 > b \qquad ...(i)$$

Let the roots of the given equation is α_1, and α_2

Then, $\alpha_1 + \alpha_2 = -2a$

$$\alpha_1\alpha_2 = b$$

$$\alpha_1 - \alpha_2 = \sqrt{(\alpha_1+\alpha_2)^2 - 4\alpha_1\alpha_2}$$

$$2m = \sqrt{4a^2 - 4b}$$

$$4m^2 = 4a^2 - 4b$$

$$m^2 = a^2 - b$$

$$b = a^2 - m^2 \qquad ...(ii)$$

From eqns (*i*) and (*ii*) b lies in the interval $[a^2 - m^2, a^2)$

50. $\begin{vmatrix} a & b & a\alpha - b \\ b & c & b\alpha - c \\ 2 & 1 & 0 \end{vmatrix} = 0,\ \alpha \neq 0$

$$a[\{(c\times 0) - 1\times(b\alpha - c)\} - b\{(b\times 0) - 2(b\alpha - c)\} + (a\alpha - b)(b\times 1 - 2c)] = 0$$

$$a(c - b\alpha) - b[-2(b\alpha - c)] + (a\alpha - b)(b - 2c) = 0$$

$$ac - ab\alpha + 2b^2\alpha - 2bc + ab\alpha - 2ac = \alpha - b^2 + 2bc = 0$$

$$2b^2\alpha - b^2 + ac - 2ac\alpha = 0$$

$$b^2(2\alpha - 1) - ac(2\alpha - 1) = 0$$

$$(b^2 - ac)(2\alpha - 1) = 0$$

$\therefore\ b^2 - ac = 0$

$\alpha \neq 0$

a, b, c are in GP.

51. We have

$$z = \frac{\alpha + i\beta}{1 - \gamma}$$

$$= \frac{\alpha}{1-\gamma} + i\frac{\beta}{1-\gamma}$$

$$\bar{z} = \frac{\alpha}{1-\gamma} - i\frac{\beta}{1-\gamma}$$

$$\frac{z - \bar{z}}{2} = i\frac{\beta}{1-\gamma} \qquad ...(i)$$

$$|z|^2 = \frac{\alpha^2}{(1-\gamma)^2} + \frac{\beta^2}{(1-\gamma)^2}$$

$$= \frac{\alpha^2 + \beta^2}{(1-\gamma)^2}$$

$$= \frac{\gamma(1-\gamma)}{(1-\gamma)^2} \qquad [\because \alpha^2 + \beta^2 = \gamma(1-\gamma)]$$

$$= \frac{\gamma}{1-\gamma}$$

$$\gamma = \frac{|z|^2}{1 + |z|^2}$$

Put in eq (*i*) we get

$$\frac{z - \bar{z}}{2i} = \frac{\beta}{1 - \frac{|z|^2}{1 + |z|^2}}$$

$$= (1 + |z|^2) \times \beta$$

$$\beta = \frac{z - \bar{z}}{2i(1 + |z|^2)} \text{ or } \frac{(\bar{z} - z)i}{2(1 + |z|^2)}$$

60. $2x + 3y + 5z = 9$

$7x + 3y - 2z = 8$

$2x + 3y + \lambda z = \mu$

Using Cramer's rule

$$D = \begin{vmatrix} 2 & 3 & 5 \\ 7 & 3 & -2 \\ 2 & 3 & \lambda \end{vmatrix} \qquad D_1 = \begin{vmatrix} 9 & 3 & 5 \\ 8 & 3 & -2 \\ \mu & 3 & \lambda \end{vmatrix}$$

$$D_2 = \begin{vmatrix} 2 & 9 & 5 \\ 7 & 8 & -2 \\ 2 & \mu & \lambda \end{vmatrix} \qquad D_3 = \begin{vmatrix} 2 & 3 & 9 \\ 7 & 3 & 8 \\ 2 & 3 & \mu \end{vmatrix}$$

System have infinitely many solutions iff

$D = 0,\ D_1 = 0,\ D_2 = 0,\ D_3 = 0$

$$D = 0, \quad \begin{vmatrix} 2 & 3 & 5 \\ 7 & 3 & -2 \\ 2 & 3 & \lambda \end{vmatrix} = 0$$

Apply $C_1 \leftrightarrow C_2$

$$\begin{vmatrix} 3 & 2 & 5 \\ 3 & 7 & -2 \\ 3 & 2 & \lambda \end{vmatrix} = 0$$

Apply $R_2 \to R_2 - R_3$

$R_3 \to R_3 - R_1$

$$\begin{vmatrix} 3 & 2 & 5 \\ 0 & 5 & -2-\lambda \\ 0 & 0 & \lambda - 5 \end{vmatrix} = 0$$

$$5(\lambda - 5) = 0$$

$$\lambda = 5$$

$$D_3 = 0. \quad \begin{vmatrix} 2 & 3 & 9 \\ 7 & 3 & 8 \\ 2 & 3 & \mu \end{vmatrix} = 0$$

Apply $C_1 \leftrightarrow C_2$

$$R_2 \rightarrow R_2 - R_3$$

$$R_3 \rightarrow R_3 - R_1$$

$$\begin{vmatrix} 3 & 2 & 9 \\ 0 & 5 & 8-\mu \\ 0 & 0 & \mu - 9 \end{vmatrix} = 0$$

$$5(\mu - 9) = 0$$

$$\mu = 9$$

System have infinitely many solutions iff $\lambda = 5$ and $\mu = 9$.

61. System has a unique solution iff $D \neq 0$

$$\begin{vmatrix} 2 & 3 & 5 \\ 7 & 3 & -2 \\ 2 & 3 & \lambda \end{vmatrix} \neq 0$$

Apply $C_1 \leftrightarrow C_2$

$$\begin{vmatrix} 3 & 2 & 5 \\ 3 & 7 & -2 \\ 3 & 2 & \lambda \end{vmatrix} = 0$$

Again Apply,

$$R_2 \rightarrow R_2 - R_3$$

$$R_3 \rightarrow R_3 - R_1$$

We get

$$\begin{vmatrix} 3 & 2 & 5 \\ 0 & 5 & -2-\lambda \\ 0 & 0 & \lambda - 5 \end{vmatrix} \neq 0$$

$$5(\lambda - 5) \neq 0$$

$$\lambda \neq 5$$

System has a unique solution iff $\lambda \neq 5$, and μ has any real value.

62. We have

$$f(x)\, f(x-1) = 1$$

$$f(x-1) = \frac{1}{f(x)}$$

Let $$I = \int_0^1 \frac{1}{1+f(x)}\,dx$$

$$= \int_0^1 \frac{1}{1+f(x-1)}\,dx \quad \left[\int_0^a f(x) = \int_0^a f(x-a)\right]$$

$$2I = \int_0^1 \frac{1}{1+f(x)}\,dx + \int_0^1 \frac{1}{1+f(x-1)}\,dx$$

$$= \int_0^1 \left[\frac{1}{1+f(x)} + \frac{1}{1+\frac{1}{f(x)}}\right] dx$$

$$= \int_0^1 \left[\frac{1}{1+f(x)} + \frac{f(x)}{1+f(x)}\right] dx$$

$$= \int_0^1 \left[\frac{1+f(x)}{1+f(x)}\right] dx$$

$$= \int_0^1 1.dx$$

$$= [x]_0^1$$

$$= 1$$

$$I = 0.5.$$

63. First row and first column number in the form of *i.e.*,

Ist $n = 0$, $\boxed{{}^0C_0} = 1$ $\boxed{1}$

IInd $n = 1$,

$^1C_0$	$^1C_1$
$^1C_1$	$^1C_0 \times {}^1C_0$

$=$

1	1
1	1

IIIrd $n = 2$,

$^2C_0$	$^2C_1$	$^2C_2$
$^2C_1$	$^2C_1 \times {}^2C_1$	$^2C_2 \times {}^2C_1$
$^2C_2$	$^2C_1 \times {}^2C_2$	$^2C_2 \times {}^2C_2$

1	2	1
2	4	2
1	2	1

Similarly,

100th, i.e., $n = 99$

Third column = $^{99}C_2$

Fourth row = $^{99}C_3$

$= {}^{99}C_2 \times {}^{99}C_3$

$= \frac{99!}{2! \times 97!} \times \frac{99!}{3! \times 96!}$

$= \frac{99 \times 98}{2} \times \frac{99 \times 98 \times 97}{6}$

$= 99 \times 49 \times 33 \times 49 \times 97$

$= 760874499.$

64. 8th layer numbers are

$n = 7,$

$^7C_0,\ ^7C_1,\ ^7C_2,\ ^7C_3,\ ^7C_4,\ ^7C_5,\ ^7C_6,\ ^7C_7$

1, 7, 21, 35, 35, 21, 7, 1

Sum of the numbers on the outermost block

$= 4[1 + 7 + 21 + 35 + 35 + 21 + 7 + 1] - 2[1 + 1]$

$= 4[128] - 4$

$= 512 - 4$

$= 508.$

65. 9th layer numbers are

$n = 8,$

$^8C_0,\ ^8C_1,\ ^8C_2,\ ^8C_3,\ ^8C_4,\ ^8C_5,\ ^8C_6,\ ^8C_7,\ ^8C_8$

1, 8, 28, 56, 70, 56, 28, 8, 1

Sum of the numbers on the diagonal block

$= 2[1 \times 1 + 8 \times 8 + 28 \times 28 + 56 \times 56 + 70 \times 70 + 56 \times 56 + 28 \times 28 + 8 \times 8 + 1 \times 1] - 70 \times 70$

$= 2[1 + 64 + 784 + 3136 + 4900 + 3136 + 784 + 64 + 1] - 4900$

$= 2[12870] - 4900$

$= 25740 - 4900$

$= 20840.$

66. 10th layer first row or fist column numbers are

$^9C_0,\ ^9C_1,\ ^9C_2,\ ^9C_3,\ ^9C_4,\ ^9C_5,\ ^9C_6,\ ^9C_7,\ ^9C_8,\ ^9C_9$

1, 9, 36, 84, 126, 126, 84, 36, 9, 1

The sum of the diagonal blocks

$= 2[1 \times 1 + 9 \times 9 + 36 \times 36 + 84 \times 84 + 126 \times 126 + 126 \times 126 + 84 \times 84 + 36 \times 36 + 9 \times 9 + 1 \times 9]$

$= 2[1 + 81 + 1296 + 7056 + 15876 + 15876 + 7056 + 1236 + 81 + 1]$

$= 2[48620]$

$= 97240.$

For Q.No. 68 to 72

Managers	Mode of Travel			Location Name			
	Rail	Car	Air	Surat	Chandigarh	Delhi	Lucknow
Shukla	✔	✘	✘	✘	✘	✘	✔
Mishra	✘	✔	✘	✔	✘	✘	✘
Singh	✘	✘	✔	✘	✘	✔	✘
Kulkarni	✘	✔	✘	✔	✘	✘	✘
Rao	✘	✘	✔	✘	✘	✔	✘
Joshi	✘	✘	✔	✘	✔	✘	✘
Nair	✘	✘	✔	✘	✔	✘	✘

73. We have,

$$x = 3\cos t$$

$$y = 4\sin t$$

We can written this as

$$\frac{x}{3} = \cos t$$

$$\frac{y}{4} = \sin t$$

$$\sin^2 t + \cos^2 t = 1$$

$$\frac{x^2}{9} + \frac{y^2}{16} = 1$$

$a = 3,\ b = 4$

$\because\ 0 < a < b$

So, this represents vertical ellipse.

75. $f(x) = \begin{vmatrix} 1+\sin^2 x & \cos^2 x & 4\sin 2x \\ \sin^2 x & 1+\cos^2 x & 4\sin 2x \\ \sin^2 x & \cos^2 x & 1+4\sin 2x \end{vmatrix}$

Apply $C_1 \to C_1 + C_2$

$$f(x) = \begin{vmatrix} 2 & \cos^2 x & 4\sin 2x \\ 2 & 1+\cos^2 x & 4\sin 2x \\ 1 & \cos^2 x & 1+4\sin 2x \end{vmatrix}$$

Apply $R_2 \to R_2 - R_1$

$R_1 \to R_1 - 2R_3$

$$f(x) = \begin{vmatrix} 0 & -\cos^2 x & -2-4\sin 2x \\ 0 & 1 & 0 \\ 1 & \cos^2 x & 1+4\sin 2x \end{vmatrix}$$

$f(x) = -(1)\ (-2 - 4\sin 2x)$

$f(x) = 2 + 4\sin 2x$

Minimum value of $\sin 2x = -1$

So, least value of $f(x) = 2 - 4 = -2$

Maximum value of $\sin 2x = 1$

So, greatest value of $f(x) = 2 + 4 = 6$

We know that period of $\sin x = 2\pi$

Period of $\sin 2x = \pi$.

79. $f(\tan\theta) = \sin^2 2\theta$

$$\int_0^1 f(x)dx = ?$$

Let, $x = \tan\theta$

$dx = \sec^2\theta\ .\ d\theta$

if $x = 0,\ \theta = 0$

$x = 1,\ \theta = \frac{\pi}{4}$

$$\int_0^{\frac{\pi}{4}} f(\tan\theta).\sec^2\theta.d\theta = \int_0^{\frac{\pi}{4}} \sin^2 2\theta.\sec^2\theta.d\theta$$

$$= \int_0^{\frac{\pi}{4}} 4\sin^2\theta\ .\ \cos^2\theta.\sec^2\theta.d\theta$$

$$= \int_0^{\frac{\pi}{4}} 4\sin^2\theta.d\theta$$

$$= 2\int_0^{\frac{\pi}{4}} (1-\cos 2\theta)d\theta \qquad [\because\ 2\sin^2\theta = 1 - \cos 2\theta]$$

$$= 2\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{4}}$$

$$= 2\left[\left(\frac{\pi}{4} - \frac{1}{2}\right) - (0-0)\right]$$

$$= \frac{\pi}{2} - 1.$$

80. We have

$$x^3 + y^3 + x^2y + 7 = 0$$

$$3x^2 + 3y^2\frac{dy}{dx} + 2xy + x^2\frac{dy}{dx} = 0$$

$$3x^2 + 2xy + \frac{dy}{dx}(3y^2 + x^2) = 0$$

$$\frac{dy}{dx}(3y^2 + x^2) = -(3x^2 + 2xy)$$

$$\frac{dy}{dx} = -\frac{(3x^2 + 2xy)}{(3y^2 + x^2)}$$

$$\left(\frac{dy}{dx}\right)_{(a,b)} = \frac{-(3a^2 + 2ab)}{(3b^2 + a^2)} \qquad ...(i)$$

Line $9x + y = 0$ passes through points (a, b) then

$$9a + b = 0$$
$$b = -9a$$

Put in eq (i)

$$\text{Tangent } \left(\frac{dy}{dx}\right)_{(a,b)} = \frac{-(3a^2 - 18a^2)}{243a^2 + a^2}$$

$$= \frac{15a^2}{244a^2}$$

$$= \frac{15}{244}.$$

85. $f(x) = x - [x + a] - b,\ ab \in R^+$

Put $x = x + 1$

$$f(x+1) = (x+1) - [x+1+a] - b$$

$$[\because [x+n] = n + [x], n \in I]$$

$$f(x+1) = x + 1 - [x+a]\,1 - b$$

$$= x - [x+a] - b$$

$$= f(x)$$

$\therefore$ Period of function $f(x)$ is '1'.

86. We have

$ab^2c^3,\ a^2b^3c^4,\ a^3b^4c^5$ are in AP

$$\therefore \quad 2 \times a^2b^3c^4 = ab^2c^3 + a^3b^4c^5$$

$$2a^2b^3c^4 = ab^2c^3\,(1 + a^2b^2c^2)$$

$$2abc = 1 + a^2b^2c^2$$

$$a^2b^2c^2 + 1 - 2abc = 0$$

$$(abc - 1)^2 = 0$$

$$abc = 1$$

We know that

$$\text{AM} \geq \text{GM}$$

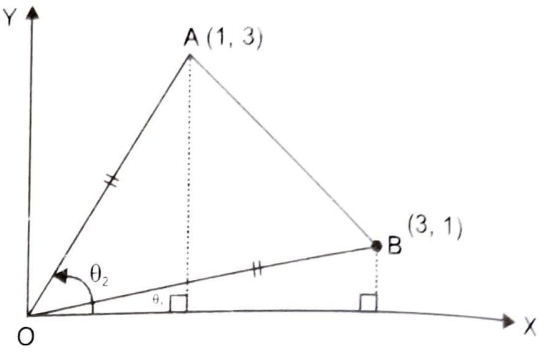
$$\frac{a+b+c}{3} \geq \sqrt[3]{a \times b \times c}$$

$$\frac{a+b+c}{3} \geq \sqrt[3]{1}$$

$$a + b + c \geq 3$$

$\therefore$ The minimum value of $a + b + c$ is 3.

91.



We know that slope $m = \tan\theta = \dfrac{\text{Length}}{\text{Base}}$

Slope of OA = 3

$$\tan\theta_1 = \frac{3}{1} = \frac{\text{Length}}{\text{Base}}$$

$\therefore$ Co-ordinate of point A is (1, 3)

Similarly,

$$\text{Slope of OB} = \frac{1}{3}$$

$$\tan\theta_2 = \frac{1}{3} = \frac{\text{Length}}{\text{Base}}$$

$\therefore$ Co-ordinate of point B is (3, 1)

Equation of line AB

$$y - 3 = \frac{1-3}{3-1}(x-1)$$

$$y - 3 = -x + 1$$

$$y = -x + 4$$

Slope $m = -1$

$\because$ Line is parallel to AB, then slope of the line is same

$$y = mx + c$$

$$y = -x + c \qquad \ldots(i)$$

$\because$ This line passes through point (–3, 7)

Then, $\quad 7 = 3 + c$

$$c = 4 \qquad \text{(Put in eq } (i))$$

Then equation is $y = -x + 4$.

92. $f(x) = 3x^{10} - 7x^8 + 5x^6 - 21x^3 + 3x^2 - 7$

$$f(1) = 3 - 7 + 5 - 21 + 3 - 7$$

$$= 11 - 35$$

$$= -24$$

$$\lim_{h\to 0}\frac{f(1-h)-f(1)}{h^3+3h}$$

$$=\lim_{h\to 0}\frac{3(1-h)^{10}-7(1-h)^8+5(1-h)^6}{\dfrac{-21(1-h)^3+3(1-h)^2-7+24}{h^3+3h}}$$

$$\lim_{h\to 0}\frac{30(1-h)^9(-1)-56(1-h)^7(-1)+30(1-h)^5(-1)}{-63(1-h)^2(-1)+6(1-h)(-1)}{3h^2+3}$$

$$=\frac{-30+56-30+63-6}{3}$$

$$=\frac{53}{3}.$$

93. (A) V # T * P

Daughter Sister

So, P is the Maternal Uncle or Maternal Aunt of V.

(B) L @ M $ R

Son Brother

So, R is the Paternal Uncle of L.

(C) D ? V # T

Mother Daughter

D is the grand-daughter or grand-son of T.

(D) M @ R $ D ? V

Son Brother Mother

M and V are husband and wife.

Hence, option (B) is correct.

94. (A) B # L # A

Daughter Daughter

A is the grand-daughter of B.

(B) M # A = N = B

Daughter Father Father

A is the grand-daughter of B.

(C) L * B = S $ Q = A

Father

Sister Father Brother Father

A is the grand-father of B.

Hence, option (C) is correct.

95. Let three number of GP are $\frac{a}{r}, a, ar$

Then,

$$\frac{a^2}{r^2}+a^2+a^2r^2=S^2 \qquad \text{Put } r^2=1$$

$$3a^2=S^2$$

$$a^2=\frac{S^2}{3}$$

$$\frac{a}{r}+a+ar=\alpha S \text{ [square both sides]}$$

$$\left(\frac{a}{r}+a+ar\right)^2=\alpha^2S^2$$

$$\frac{a^2}{r^2}+a^2+a^2r^2+2\left[\frac{a^2}{r}+a^2r+a^2\right]=\alpha^2S^2$$

[Put, $a^2=\frac{S^2}{3}$ and $r^2=1$]

$$\frac{S^2}{3}+\frac{S^2}{3}+\frac{S^2}{3}+\frac{2S^2}{3}\left[\frac{1}{r}+r+1\right]=\alpha^2S^2$$

$$S^2+\frac{2S^2}{3}\left[\frac{1+r^2+r}{r}\right]=\alpha^2S^2$$

$$1+\frac{2}{3}\left[\frac{2+r}{r}\right]=\alpha^2$$

$$1+\frac{2}{3}\left[\frac{2}{r}+1\right]=\alpha^2$$

$$1+\frac{2}{3}+\frac{4}{3r}=\alpha^2$$

$$\frac{5}{3}+\frac{4}{3r}=\alpha^2$$

$$\frac{5r+4}{3r}=\alpha^2$$

$\because \quad r^2=1$

$r=\pm 1$

$r=1,$

$$\frac{5\times 1+4}{3\times 1}=\alpha^2$$

$$\alpha^2=3$$

$$\alpha = \pm\sqrt{3}$$

(if r is strictly increasing)

If $r = -1$,

$(r < 1)$

$$\frac{-5+4}{-3} = \alpha^2$$

$$\alpha^2 = \frac{1}{3}$$

$$\alpha = \pm\frac{1}{\sqrt{3}}$$

($\because$ r is not strictly increasing)

96. $\because$ β is the root of polynomial

$x^2 + 24x + 1$

So,

$$\beta^2 + 24\beta + 1 = 0$$

$$\lim_{x\to\beta} \frac{1-\cos(x^2+24x+1)}{(x-\beta)^2}$$

$\frac{0}{0}$ Apply L'Hospital Rule

$$\lim_{x\to\beta} \frac{\sin(x^2+24x+1)(2x+24)}{2(x-\beta)}$$

$\frac{0}{0}$ Form again apply L'Hospital Rule

$$\lim_{x\to\beta} \frac{x\sin(x^2+24x+1)+12\sin(x^2+24x+1)}{x-\beta}$$

$$\lim_{x\to\beta} \frac{\sin(x^2+24x+1)+x\cos(x^2+24x+1)(2x+24)+12\cos(x^2+24x+1)(2x+24)}{1}$$

$$\lim_{x\to\beta} \sin(x^2+24x+1)+\cos(x^2+24x+1)[2x^2+24x+24x+288]$$

$$= \sin(\beta^2 + 24\beta + 1) + \cos(\beta^2 + 24\beta + 1)\, 2(\beta^2 + 24\beta + 1 + 143)$$

$$= \sin(0) + \cos(0) \times 2 \times (0 + 143)$$

$$= 0 + 1 \times 286$$

$$= 286.$$

99. Let average upto 15th inning $= x$

Then,

$$15x + 70 = 16(x + 2)$$

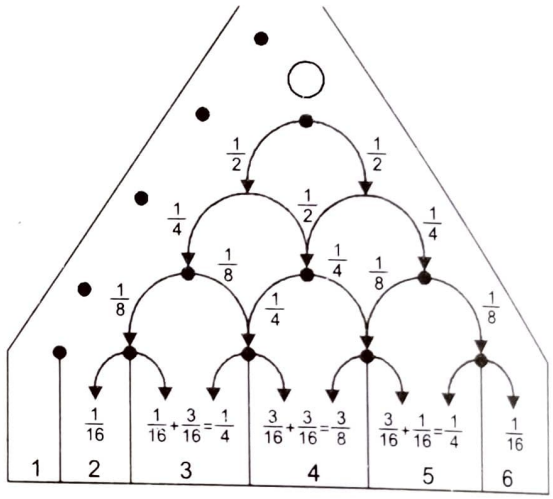
$$15x + 70 = 16x + 32$$

$$x = 70 - 32$$

$$x = 38$$

Average after 16th inning $= 38 + 2 = 40$.

100.



Probability of ball falling is

$2 \to \frac{1}{16}$

$3 \to \frac{1}{4}$

$4 \to \frac{3}{8}$

$5 \to \frac{1}{16}$

$6 \to \frac{1}{16}$

The probability of the chip falling in slot '2' is $\frac{1}{16}$.